Chapter 5. Inventory Systems

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5.1. Introduction

Inventory Definition

Material Being Held in Storage

Buffer Between and Decoupling of Two Subsequent Processes

Inventory Historical Perspective

Before 20th Century Desirable Wealth

Maximized

Early 20th Century Wasteful

Minimized

Currently an Expensive But Necessary Evil

Tradeoff or Balance

Inventory Goals and Associated Cost

The overall objective of holding inventory is to satisfy an acceptable level of customer demand. The inventory control system attempts to minimize the total system cost of providing this level of service. The total system cost traditionally consists of the costs associated with holding inventory, with ordering replenishment goods, and with unsatisfied demand. As an alternative to the cost of unsatisfied demand, a minimum service level may be used as a constraint on the inventory policy. Contemporary inventory control systems may also include the costs associated with delivering or transporting the goods from supplier to the inventory holding location and to the customer.

In traditional inventory systems the typical decisions involve the determination of the level of service to be provided, the frequency of or time between replenishments R, the order up to level *S,* and the reorder point *s*. Depending on the conditions and assumptions of the inventory system, some of these values can be parameters or variables and this has created many different types of inventory control policies. An inventory control policy is considered "optimal" if it minimizes the long-range or average total system cost. While this long-range average cost differs significantly from industry to industry and from product to product, 25 % of the product value is often used as a first approximation of the average cost for holding a product in inventory for a year. This fraction may be growing in recent years due to increasing velocity of products in the supply chain.

Inventory Justification

Economies of Scale

Production

Transportation

Response Time Constraints

Emergency

Seasonal

Required Aging Processes

Hedging Against Price Changes

Amplified by Uncertainty

Demand

Production yield and lead-time

Transportation time

Sales price and exchange rates

Inventory Tradeoffs

Tradeoff with Transportation Costs and Schedules

Tradeoff with Production or Purchasing Costs and Schedules

Tradeoff with Lost Profit and Lost Sales

Inventory Types

Pipeline Inventory

Cycle Inventory

Safety Inventory

Seasonal Inventory

Aging Inventory

Speculative Inventory

Inventory Materials

Raw Materials

Incoming to Organization

Work in Process

Internal to Organization

Finished Goods

Outgoing from Organization

Inventory Questions

What to Store

Where to Store

When to Place an Order

Inventory Costs

All inventory calculations in the following inventory models and policies are expressed in economic terms and values for the organization holding and owning the inventory.

The *unit cost* or *unit value* ($/Unit) represents the cost invested so far in a single unit of the product. This cost includes the total production cost, purchasing cost, transportation cost and any applicable duties. The unit value is not a direct inventory cost but is one of the factors in the inventory holding cost.

The *holding cost* or *carrying cost*, expressed as a cost per unit of inventory per unit of time ($/Unit-Year), represents all the costs associated with storing the inventory. This costs includes the costs associated with storage facilities, handling, insurance, pilferage, obsolescence, and the opportunity cost of capital. The *holding cost rate* is the ratio of the holding cost divided by the value of one unit of inventory ($/$-Year). A holding cost rate of 25 % of the unit value per year is a widely quoted average for the US industry.

The *reorder cost*, also called the fixed ordering cost or setup cost ($/Order) represents all the costs incurred each time an order is placed. This cost includes the costs associated with preparing the purchase, receiving the order, paying the invoice, with equipment setups, or with transporting the order.

The *shortage cost*, expressed as a cost per unit of inventory ($/Unit) represents all the costs associated with the delays and remedial actions when inventory was not available to satisfy customer demand. This costs includes emergency transportation costs, penalties, and loss of customer goodwill. The shortage cost is the most difficult to estimate accurately of the three inventory costs, especially the components related to loss of customer goodwill or the cost of lost sales of other items in the same customer order.

Inventory Performance Measures

Service Level

Inventory systems are expected to satisfy an acceptable level of customer demand known as the service level. Three types of service levels are commonly used:

1. The α, P1, or type 1 service level is defined as the probability of not being out of stock per replenishment cycle. It is equal to the long-range fraction of cycles in which a stockout does not occur. This service level is also called the cycle service level. This service level measures whether or not a backorder occurs but is not concerned with the size of the backorder.

2. The β, P2, or type 2 service level is defined as the long-range average fraction of demand delivered from inventory on the shelf. This service level is also called the *fill rate*. This service level considers not only the probability of a stockout but also the size of the backorder.

3. There exist several different definitions of the third service level. The γ, P3, or type 3 service level is defined in Silver et al. (1998) as the fraction of the time during which there is inventory present on the shelf. This definition of the service level is also called the *ready rate*. Schneider (1981) defines the $(1-\gamma)$ service level as the ratio of the long run average cumulative unsatisfied demand per replenishment cycle divided by the average demand per replenishment cycle.

The first type of service level measures if there is a stockout, the second service level measures the size of the stockout, and the third service level measures the size and the duration of the stockout. Since higher types of service level compute a more refined and accurate measure of customer service, it should come as no surprise that the determination of the optimal inventory policy becomes much more difficult when moving from the use of type α to β to γ. In addition, identical numerical values for α and β and γ indicate significantly different levels of customer service.

Consider the on-hand inventory level over four replenishment cycles as shown in Figure 5.1. Each replenishment cycle is 10 time periods long and the average demand per period is 4 units or 40 units per cycle.

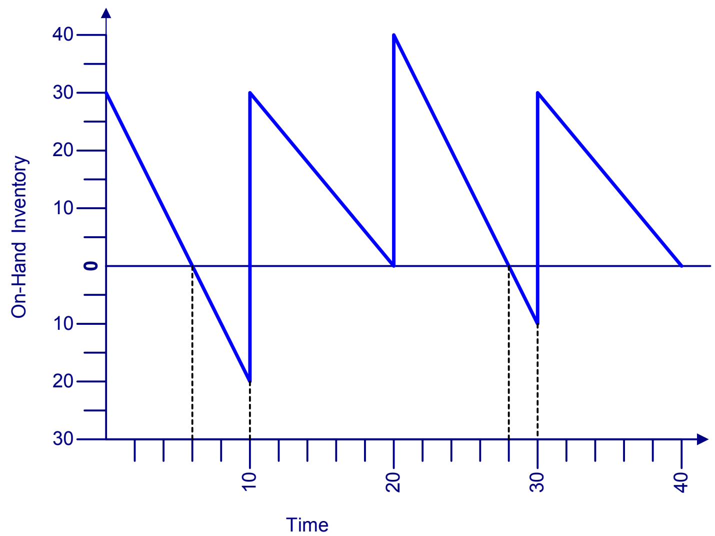


Figure 5.1. On-Hand Inventory Levels Illustration

Two cycles out of the four experience backorders. While considering only four cycles is clearly not a long run average, the α service level based on these four cycles is computed as:

$$\alpha = 1 - \frac{2}{4} = 0.5$$

The first cycle experiences 20 units backorder and the third cycle experiences a backorder of 10 units. The second and fourth cycles have no backorders. The β service level is then computed as:

$$\beta = 1 - \frac{20+10}{160} = 0.81$$

The cumulative amount of backordered units over time can be computed by measuring the backorder quantities at the end of each period. In the above example the backorder quantities are 5, 10, 15, 20 (in period one) and 5, 10 (in period three). The cumulative amount of backordered units over time can then be computed as

$$\gamma = 1 - \frac{(5+10+15+20+5+10)/40}{4} = 0.59$$

A continuous approximation of the cumulative amount of backordered units over time can be computed as the area below the zero inventory level, or for the above example the areas of the two triangles below the zero inventory level

$$\gamma = 1 - \frac{(40+10)/40}{4} = 0.69$$

The difference between those two values is caused by the small time offset when the backorder quantities are measured in each period. The continuous approximation measures the backorder amounts at the midpoint of each period, where the discrete computation measures the backorder amounts at the end of each period. The difference is 2.5 units backordered for the six periods with backorders, or exactly the difference between 50 and 65 in the numerators of the above formulas.

Finally, the ready rate for this example is computed as

$$\gamma = 1 - \frac{6}{40} = 0.85 .$$

Stock Turnover

The stock turnover indicates the speed with which a single product moves through the distribution system. It is expressed in turns per year and is computed as the ratio of the value of the annual sales divided by the value of the average inventory of that product. A higher level of stock turnover is more desirable since it indicates that more sales have been made for the amount of inventory in the system.

The average time in inventory for a product indicates on the average how long an item of the product is kept in inventory before it is sold. The average time in inventory is the inverse measure of the stock turnover and it is expressed in years.

Basic Inventory Policy Classes

An inventory policy basically provides the answer to two related questions when managing a product inventory: when and how much to order. The first answer determines how often the inventory levels are checked and when orders are placed. The second answer determines how much inventory is ordered, which is called the order quantity. Depending on the answer to the above two questions, inventory policies can be divided into three classes.

The first class contains inventory policies that always order the same quantity. These are called *fixed order quantity policies*. These policies adjust the time between order placements depending on the demand. They are often used for products that have a small or irregular demand or for products that have significant savings for being replenished in a particular quantity. The second class contains inventory policies that check the current inventory at regular time intervals and place an order if needed.

These are called *fixed order frequency policies*. These policies adjust the order quantity, which may be zero. They are often used for item with high and regular demand.

Direct Demand Satisfaction

Adjust quantity and time

Push or Pull

Inventory Trends

Increase handling over inventory

Increase transportation over handling and inventory

Increase information over inventory and handling

5.2. Independent Demand Systems

Deterministic Demand

Pipeline Inventory

In-Transit or Aging & Curing Inventory

PI	pipeline inventory
D	demand during the planning period (units / year)
TT	transit time in planning periods (years)

$$PI = D \cdot TT \tag{5.1}$$

Cycle Inventory

Transition Between Regular Input and Output Quantities (Batch Sizes)

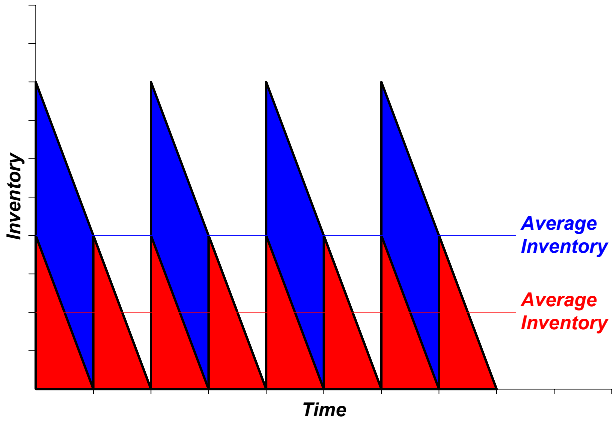


Figure 5.2. Cycle Inventory Illustration for Different Cycle Lengths

CI	cycle inventory
d	period demand rate (units / period)
ct	cycle time (periods)
CT	cycle time (years)
D	demand rate (units / year)

$$CI = \frac{D \cdot CT}{2} = \frac{d \cdot ct}{2} \tag{5.2}$$

Seasonal Inventory

Seasonal inventory is created during periods of lower demand and distributed during periods of high demand. This allows an organization to use a lower installed production capacity and still to meet all the customer demands. The reduced production capacity must be traded off with the inventory holding costs from period to period.

The following assumptions are usually made. The periods in the cycle have equal duration. The value of the product does not change from period to period. The inventory buildup or withdrawal occurs on the average at a constant rate, which is the difference between the constant production and constant demand rate for that period. This implies that in each period, the inventory change is linear. The inventory levels in a seasonal system are illustrated in Figure 5.3.

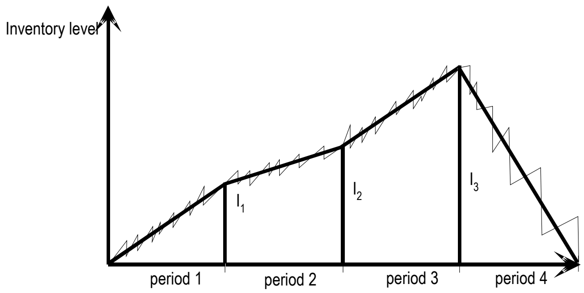


Figure 5.3. Seasonal Inventory Levels

Let I_t be the inventory level at the end of period *t* and I_0 indicates the initial inventory. Let *v* be the value of the product and let *h* be the holding cost rate expressed as a percentage of the value of the product for holding the product in inventory for one period, i.e. expressed in dollars per dollars per period.

For cyclical systems, where the ending inventory of the last period in the cycle is equal to the starting inventory of the first period, the seasonal inventory holding cost is given by

$$SIC = hv\left(\sum_{t=1}^{N} I_t\right) \tag{5.3}$$

If the system is not cyclical, the seasonal inventory holding cost is given by

$$SIC = hv\left(\frac{I_1}{2} + \sum_{t=2}^{N-1} I_t + \frac{I_N}{2}\right) \tag{5.4}$$

Finally, if the periods are not of equal length, then h_i is the holding cost rate for period *i* and the seasonal inventory cost is given by

$$SIC = v\sum_{t=1}^{N} \frac{h_t\left(I_{t-1} + I_t\right)}{2} \tag{5.5}$$

Economic Order Quantity (EOQ)

One of the simplest logistics systems is the system in which the demand is constant. The models for constant demand can also be applied in situations with predictable demand, where the forecast error and uncertainty is small. Often there is fixed setup cost for production or order, which has to be balanced against an inventory holding cost which grows proportionally to the amount held in inventory. The economic order quantity finds the best tradeoff between the fixed and the variable cost assuming there is a constant demand rate.

$$
\begin{aligned}
TC(Q) &= IC + OC \\
&= \frac{hc \cdot Q}{2} + \frac{D}{Q} oc
\end{aligned}
\tag{5.6}
$$

$$
\begin{aligned}
&\frac{d(TC)}{dQ} = \frac{hc}{2} - \frac{D \cdot oc}{Q^2} \\
&\frac{d^2(TC)}{dQ^2} = \frac{2 \cdot D \cdot oc}{Q^3} > 0 \\
&\left.\frac{d(TC)}{dQ}\right|_{Q \to 0} = -\left.\frac{D \cdot oc}{Q^2}\right|_{Q \to 0} = -\infty \\
&\frac{d(TC(Q^*))}{dQ} = \frac{hc}{2} - \frac{D \cdot oc}{Q^{*2}} = 0 \\
&Q^{*2} = \frac{2 \cdot D \cdot oc}{hc}
\end{aligned}
\tag{5.7}
$$

$$
Q^* = \sqrt{\frac{2 \cdot D \cdot oc}{hc}}
\tag{5.8}
$$

$$
\begin{aligned}
TC(Q^*) &= \frac{hc}{2}\sqrt{\frac{2 \cdot D \cdot oc}{hc}} + \frac{D \cdot oc}{\sqrt{\frac{2 \cdot D \cdot oc}{hc}}} \\
&= \sqrt{2 \cdot D \cdot oc \cdot hc}
\end{aligned}
\tag{5.9}
$$

Example

Microsoft Excel - Inventory EOQ Variants Harvey ...

	A	B	C	D
13	Purchase Price	p	10	$/unit
14	Lead Time	LT	0.5	year
15	Holding Cost Rate	hcr	0.2	/year
16	Shortage Cost	sc	25	$/unit
17	Ordering Cost	oc	50	$/order
18	Demand Rate	d	200	units/year
19	SD Demand Lead Time	sdlt	25	units
20	Probability No-Stockout	F(R)	0.98	
21	Fill Rate	fr	0.98	
22	SD Lead Time	slt	0.125	years
23				
24	Mean Demand Lead Time	dlt	100	units
25	SD Demand	sd	35.4	units

Figure 5.4. Inventory Example Data

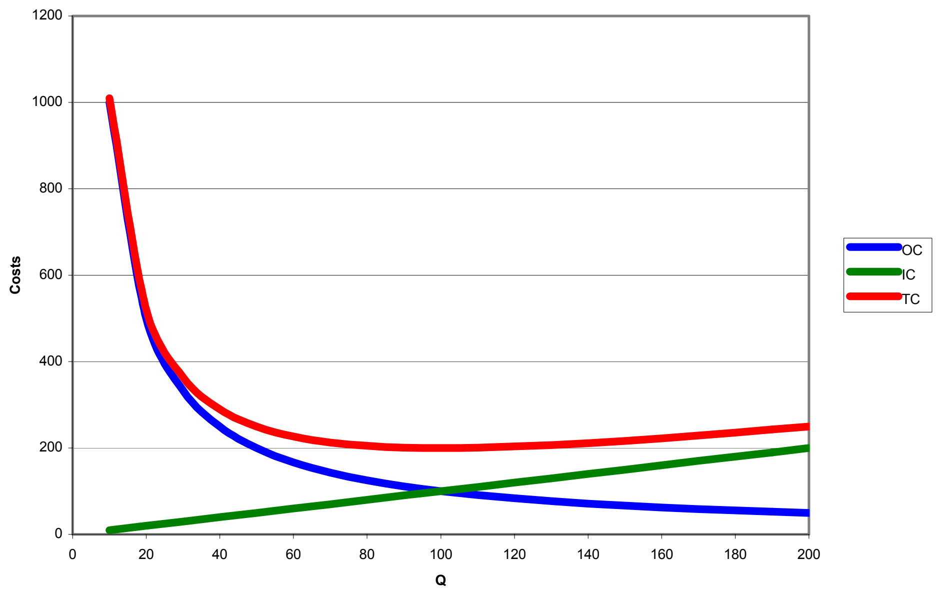


Figure 5.5. Inventory Example Cost Curves for Known Demand

$$Q = \sqrt{\frac{2 \cdot 200 \cdot 50}{0.2 \cdot 10}} = 100$$

$$\begin{aligned} TC &= \frac{100 \cdot 0.2 \cdot 10}{2} + \frac{200}{100} 50 \\ &= 200 \end{aligned}$$

Economic Production Quantity

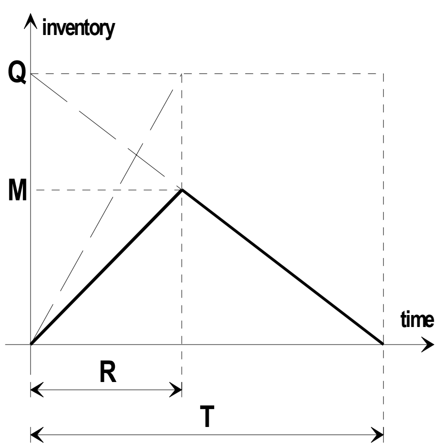


Figure 5.6. Inventory Pattern for Finite Production and Demand Rates

$$M = Q - d \cdot R = Q - d\left(\frac{Q}{p}\right) = Q\left(1 - \frac{d}{p}\right) \tag{5.10}$$

$$
\begin{aligned}
TC &= IC + MC \\
&= \frac{HC \cdot M \cdot T}{2} + FC + VC \cdot Q \\
&= \frac{HC \cdot Q \cdot (1 - d/p) \cdot Q}{2 \cdot d} + FC + VC \cdot Q \\
&= FC + VC \cdot Q + \frac{HC(p-d)}{2 \cdot p \cdot d} Q^2
\end{aligned} \tag{5.11}
$$

$$
tc = \frac{TC}{Q} = \frac{FC}{Q} + VC + \frac{HC(p-d)}{2pd} Q \tag{5.12}
$$

$$
\frac{d(tc)}{dQ} = -\frac{FC}{Q^2} + \frac{HC(p-d)}{2pd} Q = 0 \tag{5.13}
$$

$$
\frac{d^2(tc)}{dQ^2} = \frac{2 \cdot FC}{Q^3} + \frac{HC \cdot (p-d)}{2pd} > 0 \tag{5.14}
$$

$$
Q^* = \sqrt{\frac{2 \cdot FC \cdot d}{HC(1 - \frac{d}{p})}} \tag{5.15}
$$

$$
TC^* = 2 \cdot FC + VC \sqrt{\frac{2pd \cdot FC}{HC(p-d)}} \tag{5.16}
$$

Production Batch Size Example

From the marketing and sales department point of view, the best batch size is equal to one. This is equivalent to a make-to-order policy or demand driven production and gives the sales department the greatest flexibility. However, such small batch sizes might not be efficient for the manufacturing department if there are significant setup costs. For example, a customer could walk into a car dealership, "assemble" his own car from the available options. This would be acceptable to the customer with a manufacturing lead-time of one week and a delivery lead time of a week. Current realistic lead times for this scenario are much longer. Business corporations have identified the capability to manufacture on demand rather than to inventory as a major competitive advantage. This manufacturing philosophy is called "mass customization".

An example of either extreme point of the spectrum of manufacturing technology is given next. Henry Ford is attributed the quote that “the customer could order a car in any color he desired, as long it was black” illustrating the state of the art in the automotive assembly process of the model T. This statement is a reflection that a single product is easier and more efficient to manufacture. On the other hand, captain Jean-Luc Picard of the starschip enterprise in “Star Trek: the Next Generation” can order a single cup of strong tea which is immediately delivered by the replicator in his quarters. This level of manufacturing flexibility and efficiency only exists in science fiction.

In computing the optimal batch size in production operations from the manufacturing point of view, we will use the following definitions:

T = Production cycle which repeats indefinitely.

R = Production time for the product being evaluated

Q = Production batch size for this product

M = Maximum inventory of this product during the production cycle

d = Product demand rate

p = Product production rate (p > d)

IC = Total inventory cost

MC = Total manufacturing cost

TC = Total cost

HC = Inventory holding cost per cycle per product unit

FC = Fixed costs for starting production of a batch of this product

VC = Variable (marginal) cost for production of one unit of this product

ic = Inventory cost per unit

mc = Manufacturing cost per unit

tc = Total unit cost

The inventory pattern over a cycle T is given in the next Figure.

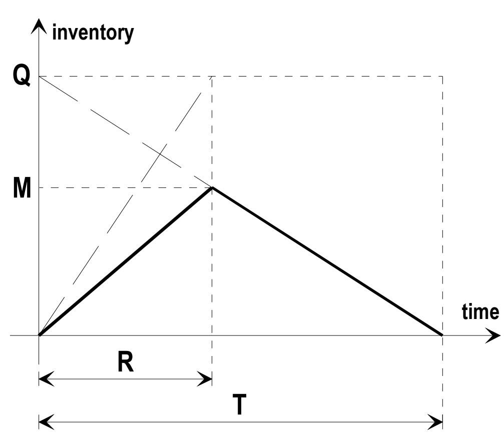


Figure Error! Bookmark not defined.**.7. Inventory Pattern in Production Systems**

We first compute the maximum product inventory:

$$M = Q - d \cdot R = Q - d\left(\frac{Q}{p}\right) = Q\left(1 - \frac{d}{p}\right) \tag{5.17}$$

Next we compute the total costs over a full cycle:

$$\begin{aligned} TC &= IC + MC \\ &= \frac{HC \cdot M \cdot T}{2} + FC + VC \cdot Q \\ &= \frac{HC \cdot Q \cdot (1 - d/p) \cdot Q}{2 \cdot d} + FC + VC \cdot Q \\ &= FC + VC \cdot Q + \frac{HC(p-d)}{2 \cdot p \cdot d} Q^2 \end{aligned} \tag{5.18}$$

We then compute the unit costs:

$$tc = \frac{TC}{Q} = \frac{FC}{Q} + VC + \frac{HC(p-d)}{2pd} Q \tag{5.19}$$

We find the optimal batch size by setting the first derivative equal to zero:

$$\frac{d(tc)}{dQ} = -\frac{FC}{Q^2} + \frac{HC(p-d)}{2pd} Q = 0 \tag{5.20}$$

$$Q^* = \sqrt{\frac{2 \cdot FC \cdot d}{HC(1 - \frac{d}{p})}} \tag{5.21}$$

This is a generalization of the standard "Economic Order Quantity" or EOQ formula for which the production rate is infinite. The optimal batch size can then also be called the "Economic Production Quantity" or EPQ for a finite production rate.

The optimal total cost is then given by:

$$\begin{aligned} TC^* &= \frac{HC(p-d)\left(\frac{2pd \cdot FC}{HC(p-d)}\right)}{2pd} + FC + VC\sqrt{\frac{2pd \cdot FC}{HC(p-d)}} \\ &= 2 \cdot FC + VC\sqrt{\frac{2pd \cdot FC}{HC(p-d)}} \end{aligned} \tag{5.22}$$

Most of the factors in this equation are beyond the control of the production system. For example, the inventory holding cost HC is determined by the cost of capital and storage in the facility. The only way to reduce the optimal, efficient batch size is then to reduce the fixed or setup cost.

The minimization by taking the first derivative and setting it to zero is valid since the second derivative is positive and this proves that tc is convex with respect to Q.

$$\frac{d^2(tc)}{dQ^2} = \frac{2 \cdot FC}{Q^3} + \frac{HC \cdot (p-d)}{2pd} > 0 \qquad (5.23)$$

Note that FC(Q*) = IC(Q*) and fc(Q*) = ic (Q*).

Economic Order Quantity with Replenishment Lead Time

Deterministic Demand (D) and Demand rate (d)

Deterministic Lead Time (LT)

Reorder Point = Quantity On Hand when Order is Placed

$$R = d \cdot LT \qquad (5.24)$$

If the lead time is longer than the replenishment cycle then the reorder point will be larger than the order quantity, which is equal to the maximum on-hand inventory. Two alternative interpretations can then be used.

1. Place an order when the sum of the on-hand and on-order inventory falls below the reorder point
2. Place an order when the on-hand inventory falls below the reorder point modulo the order quantity. If the reorder point is an integer multiple of the order quantity, then this modulo operation yields zero and an order must be placed when the on-hand inventory falls below the order quantity.

$$Q = \sqrt{\frac{2 \cdot 200 \cdot 50}{0.2 \cdot 10}} = 100$$

$$TC = \frac{100 \cdot 0.2 \cdot 10}{2} + \frac{200}{100} 50 = 200$$

$$R = 200 \cdot 0.5 = 100$$

Stochastic Demand

In most logistics systems, the future demand is not known with complete certainty but rather exhibits a significant amount of randomness. In order to use scientific methods for the determination of the inventory, some information must be known about the future demand. Typically, either the complete demand distribution or at least the expected demand and the standard deviation of the demand are assumed to be known.

Different inventory ordering policies are used depending on the fact if ordering occurs once or repetitively.

Single Order Inventory Policies

The inventory policies of this section apply to logistics systems where there exists a single opportunity to order inventory to satisfy some future demand during a finite sales period. The demand is unknown at the time of ordering but its cumulative demand distribution is known. After the sales period has completed, there is no further demand for the product. Examples are the ordering of daily newspapers by the owner of a newspaper stand, ordering of perishable fruit and flowers by a street vendor, ordering of Christmas trees for a charity fund raiser, and the ordering of fashion products with particular styles and colors. In each case the sole decision to be made is how much inventory to order. Each unit of inventory ordered has a constant purchase price. If not enough inventory has been ordered, a shortage occurs and the vendor could have sold more units during the sales period and made more profit. The potential extra profit per product unit is called the *shortage cost*, *underage cost*, or *marginal profit*. If too much inventory has been ordered, the vendor is left with excess units after the sales period and these units must be disposed of at a reduced rate, which is called the *salvage value*. The salvage value may even be negative, indicating that the vendor has to pay a third party to dispose of the remaining units. The difference between the purchase price and the salvage value is called the *excess cost*, *overage cost*, or *marginal loss*. It is the objective of the vendor to order the amount of inventory that minimizes the sum of the expected shortage and excess costs. This amount corresponds to the optimal tradeoff between the cost disposing excess items and the loss in profit caused by shortages. There exist many applications in logistics systems with these characteristics, and as a consequence this problem has been studied extensively. It has been traditionally called the *Newsboy Problem* and is currently known as the *Newsvendor Problem*.

We will use the following notation

Q	amount of inventory purchased (units)
D	actual (unknown) demand during the single period (units)
$f(x)$	demand distribution
$F(x)$	cumulative demand distribution
$\bar{d}$	expected demand during the single period (units)

p sales price ($/unit)

c purchase price ($/unit)

s salvage value ($/unit)

The shortage cost (c_s) or marginal profit and the excess cost (c_e) or marginal loss are then computed as:

$$\begin{aligned} c_s &= p - c \qquad if\ Q \le D \\ c_e &= c - s \qquad if\ Q > D \end{aligned} \tag{5.25}$$

To obtain realistic results, the cost parameters must satisfy the following sequence of inequalities, or equivalently, the shortage cost and excess cost must have positive values.

$$p > c > s \tag{5.26}$$

If the salvage value were not smaller than the purchase price ($s \ge c$), then the vendor would purchase an infinite inventory, since all unsold items can be disposed of without a loss. Similarly, if the purchase price were not smaller than the sales prices ($c \ge p$), then the vendor would not purchase any inventory, since each item purchased and sold would cost the vendor money.

The expected sum of shortage and excess cost is given by

$$G(Q) = c_e \int_0^Q (Q-x) f(x) dx + c_s \int_Q^\infty (x-Q) f(x) dx \tag{5.27}$$

To compute the derivative of $G(Q)$ we use Leibniz's rule, which states that

$$\frac{d}{dy} \int_{l(y)}^{u(y)} h(x,y) dx = \int_{l(y)}^{u(y)} \left(\frac{\partial h(x,y)}{\partial y} \right) dx + h(u(y),y) \frac{du(y)}{dy} - h(l(y),y) \frac{dl(y)}{dy} \tag{5.28}$$

This yields

$$\begin{aligned} \frac{dG(Q)}{dQ} &= c_e \int_0^Q f(x) dx + c_s \int_Q^\infty -f(x) dx \\ &= c_e F(Q) - c_s (1 - F(Q)) \end{aligned} \tag{5.29}$$

In order for the optimal value of *Q* to be found where the first derivative equals zero, the first derivative must be negative for an extreme left value of *Q,* such as $Q = 0$, and the second derivative must be positive everywhere, indicating that the first derivative is increasing over the full region of *Q*.

$$\frac{d^2G(Q)}{dQ^2} = (c_s + c_e) f(Q) \geq 0 \qquad \forall Q$$
$$\left.\frac{dG(Q)}{dQ}\right|_{Q=0} = c_e F(0) - c_s(1 - F(0)) = -c_s < 0 \tag{5.30}$$

Since $G(Q)$ satisfies these two conditions, its optimal value can be found by setting its first derivative equal to zero.

$$\frac{dG(Q^*)}{dQ} = (c_e + c_s)F(Q^*) - c_s = 0$$
$$F(Q^*) = \frac{c_s}{c_e + c_s} \tag{5.31}$$
$$Q^* = F^{-1}\left(\frac{c_s}{c_e + c_s}\right)$$

Equilibrium theory states that at the optimal inventory level the expected profit of selling one more item equals the expected loss of one item of excess inventory

$$c_s(1 - F(Q^*)) = c_e F(Q^*)$$
$$F(Q^*) = \frac{c_s}{c_e + c_s} = \frac{p - c}{p - s} \tag{5.32}$$

This is equivalent to stating that the optimal type-one service level for this system is again given by equation (5.32), where the type-one service level gives the probability that all demand in a period is satisfied immediately from stock.

If an additional cost (π) were associated with each unit of unsatisfied demand, this cost would be added to the shortage cost in expression (5.32). Additional costs may be caused by the loss of customer goodwill.

$$F(Q^*) = \frac{c_s}{c_e + c_s} = \frac{\pi + p - c}{\pi + p - s} \tag{5.33}$$

The optimal service level and corresponding optimal inventory can also be determined by optimizing the expected profit. The profit achieved is a function of the quantity ordered before the sales period starts (Q) and the actual demand observed during the sales period (D). The expression for the profit is then:

$$Profit(Q,D)=\begin{cases}(p-c)Q & Q\le D\\ pD-cQ+s(Q-D) & Q>D\end{cases} \tag{5.34}$$

$$E\left[Profit(Q)\right]=p\int_0^Q x\cdot f(x)\,dx+s\int_0^Q (Q-x)\cdot f(x)\,dx+p\int_Q^\infty Q\cdot f(x)\,dx-cQ \tag{5.35}$$

The first two terms compute the revenue and salvage when the demand does not exceed the purchased supply of items; the third term computes the revenue when the demand exceeds the supply. In either case, the cost of items purchased, indicated by the fourth term $-cQ$, must be subtracted of the expected profit. The only unknown in the expression of the expected profit is the quantity of items purchased. To find the optimal quantity, we compute the derivative using again Leibnitz's rule.

$$\frac{dE\left[Profit(Q)\right]}{dQ}=s\cdot F(x)+p\cdot\left[1-F(x)\right]-c$$

Setting the derivative equal to zero and rearranging the term yields again (5.32).

If the cumulative demand distribution has discrete breakpoints, the optimal Q^* is chosen so that the cumulative demand is no smaller than the computed ratio, i.e., we find the Q^* by rounding up.

If we assume that the demand is normally distributed, the optimal inventory level can be found based on tables of the Normal distribution or using a spreadsheet with its built in functions for the inverse Normal distribution or the inverse standard Normal distribution. Similar computations can be made for other continuous demand distributions, provided we can compute the mean, standard deviation, and inverse cumulative distribution.

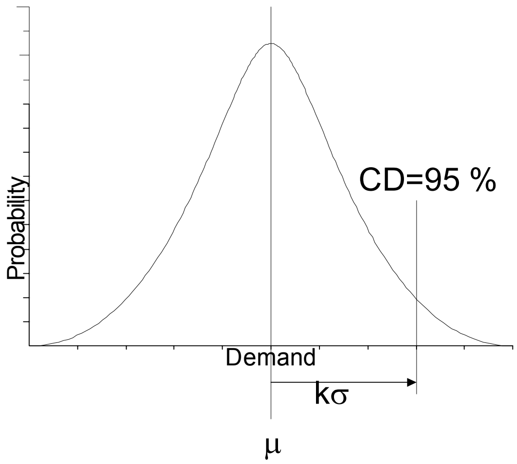


Figure 5.8. Normal Distribution

$$k^* = N^{-1}\left(\frac{c_s}{c_s + c_e}\right)$$
$$Q^* = \overline{d} + k^* \sigma_D \qquad (5.36)$$

Sports Team Championship T-shirt Vendor Example

The Cowpunchers are a football team that perennially wins the league championship title. A street vendor of championship T-shirts must order the T-shirts three days before game day and pay a purchase price of \$5. The shirts can be sold for \$15 on game day at the exit gate of the stadium. Based on his sales experience during the previous years, the vendor, who is a Georgia Tech student moonlighting as a street vendor, estimates that the average demand will be 100 shirts, is normally distributed, and that the standard deviation of the demand is 30 shirts. After the game day, the vendor returns to attending classes and donates all unsold shirts to charity.

The cumulative demand distribution at the optimal inventory level Q^* equals

$$F(Q^*) = \frac{(15-5)}{(15-5)+(5-0)} = \frac{10}{15} = 0.667$$

$$k^* = N^{-1}(0.667) = 0.431$$

$$Q^* = \overline{d} + k^* \sigma = 100 + 0.431 \cdot 30 = 112.9 \approx 113$$

The vendor should order 113 shirts.

If he could sell any remaining shirts for \$3 per shirt to another street vendor, who will continue selling the shirts on a street corner after game day, then his optimal inventory would increase to

$$F(Q^*) = \frac{(15-5)}{(15-5)+(5-3)} = \frac{10}{12} = 0.833$$

$$k^* = N^{-1}(0.833) = 0.967$$

$$Q^* = \overline{d} + k^*\sigma = 100 + 0.967 \cdot 30 = 129.0 \approx 129$$

How to test if a distribution is the Normal distribution

The chi-square Goodness of Fit test was originally proposed by Pearson in 1900 to test if an observed frequency distribution conforms to another theoretical distribution by comparing the frequencies of experimental observations to the expected frequencies of the theoretical distribution. The frequencies in any of the intervals should not be less than 5. If this condition is not satisfied, several intervals can be combined to create frequencies not less than 5. The Kolmogorov-Smirnov goodness-of-fit test is preferred if the frequencies in the intervals are small and it can be used for very small frequencies, where the chi-square test does not apply. Hence, the Kolmogorov-Smirnov test is more powerful than the chi-square goodness-of-fit but it cannot be used for discrete distributions. We are interested in establishing if the experimental data is conformant to the continuous normal distribution, so both test can be used if the frequencies are larger than 5. The following notation is used for the chi-square test

f_i	experimental frequency of interval *i*
π_i	theoretical probability of interval *i*
N	number of observations
K	number of intervals

The chi-square test statistic is computed as

$$\chi^2 = \sum_{i=1}^{K} \frac{\left(f_i - N\pi_i\right)^2}{N\pi_i} \tag{5.37}$$

The computed chi-square has the value of zero for a perfect fit and is large when the fit is bad. If the computed chi-square value is smaller than the α percentile of the chi-square distribution with $K-1$ degrees of freedom, then the null hypothesis that the data has the theoretical distribution cannot be rejected with an α level of significance. Recall that the level of significance is the probability of a type-one error, i.e. the probability of erroneously rejecting the null hypothesis.

Several other less rigorous tests can be used to investigate if the experimental data are conformant to a theoretical normal distribution. If any of the tests fail, then the experimental data is not conformant to a normal distribution. If the data pass all the tests below, then the chi-square goodness of fit test can be used to test conformity with a prescribed level of confidence. The tests on the experimental data are:

The coefficient of variation is less than 0.5.

The probability of a negative demand is less than 2 %.

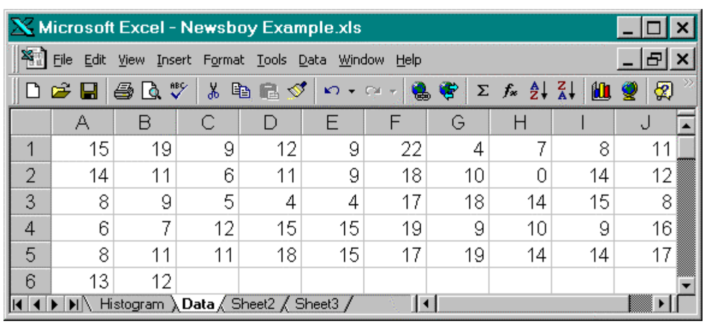
The probability that the demand is more than the mean plus two standard deviations is less than 2 %.

The corresponding formulas are:

$$
\begin{aligned}
& CV = \frac{\sigma}{\mu} \leq 0.5 \\
& P[x < 0] = F(0) \leq 0.02 \\
& P[x > \mu + 2\sigma] = 1 - F(\mu + 2\sigma) \leq 0.02
\end{aligned}
\tag{5.38}
$$

Newsstand Vendor Example

The newsvendor has diligently recorded the number of customers that attempted to purchase the weekly edition of a news magazine for the last 52 weeks. This includes both the number of magazines sold and the number of unsatisfied customers because of insufficient number of magazines in inventory. The mean demand was 11.73 and the standard deviation of the demand was 4.74.



	A	B	C	D	E	F	G	H	I	J
1	15	19	9	12	9	22	4	7	8	11
2	14	11	6	11	9	18	10	0	14	12
3	8	9	5	4	4	17	18	14	15	8
4	6	7	12	15	15	19	9	10	9	16
5	8	11	11	18	15	17	19	14	14	17
6	13	12								

Figure 5.9. Single Order Historical Sales Records

The vendor can sell the magazine for 75 cents per copy. Each copy that he purchases costs him 25 cents. He can sell all unsold copies back to the publisher for 10 cents per copy. The marginal costs per copy and the optimal service level are computed next.

$$c_s = 75 - 25 = 50$$

$$c_e = 25 - 10 = 15$$

$$F\left(Q^*\right) = \frac{50}{50+15} = 0.77$$

Computing the cumulative relative frequencies and sorting them by increasing demand and determining the smallest demand for which the cumulative frequency equals or exceeds the optimal service level finds the smallest inventory value for which the cumulative demand satisfied out of inventory is larger than the optimal type-1 service level. The next figure displays the frequency, relative frequency, and cumulative relative frequency in function of the demand quantity for the example. The smallest demand with cumulative frequency larger than 0.77 equals 15 units.

Microsoft Excel - Newsboy Example.xls

	B	C	D	E
1	*Bin*	*Frequency*	Rel Freq	*Cum Freq*
2	0	1	0.01923	0.0192
3	1	0	0.00000	0.0192
4	2	0	0.00000	0.0192
5	3	0	0.00000	0.0192
6	4	3	0.05769	0.0769
7	5	1	0.01923	0.0962
8	6	2	0.03846	0.1346
9	7	2	0.03846	0.1731
10	8	4	0.07692	0.2500
11	9	6	0.11538	0.3654
12	10	2	0.03846	0.4038
13	11	5	0.09615	0.5000
14	12	4	0.07692	0.5769
15	13	1	0.01923	0.5962
16	14	5	0.09615	0.6923
17	15	5	0.09615	0.7885
18	16	1	0.01923	0.8077
19	17	3	0.05769	0.8654
20	18	3	0.05769	0.9231
21	19	3	0.05769	0.9808
22	20	0	0.00000	0.9808
23	21	0	0.00000	0.9808
24	22	1	0.01923	1.0000
25	More	0	0.00000	
26		52	1.00000	

Figure 5.10. Single Order Frequency Distribution of Sales

The frequency distribution of the experimental data and of the theoretical normal distribution with the same mean and standard deviation values is shown in the next figure. Since the observed frequencies are well below five or even zero in several intervals when unit intervals are used, the chi-square goodness-of-fit test cannot be used. The maximum deviation between the cumulative distribution of the experimental data and the normal distribution is 0.0516. The critical value of the Kolgomorov-Smirnov distribution for $N = 52$ and $\alpha = 0.01$ is 0.23. The null hypothesis that the demand is normally distributed cannot be rejected at the 0.01 level of significance.

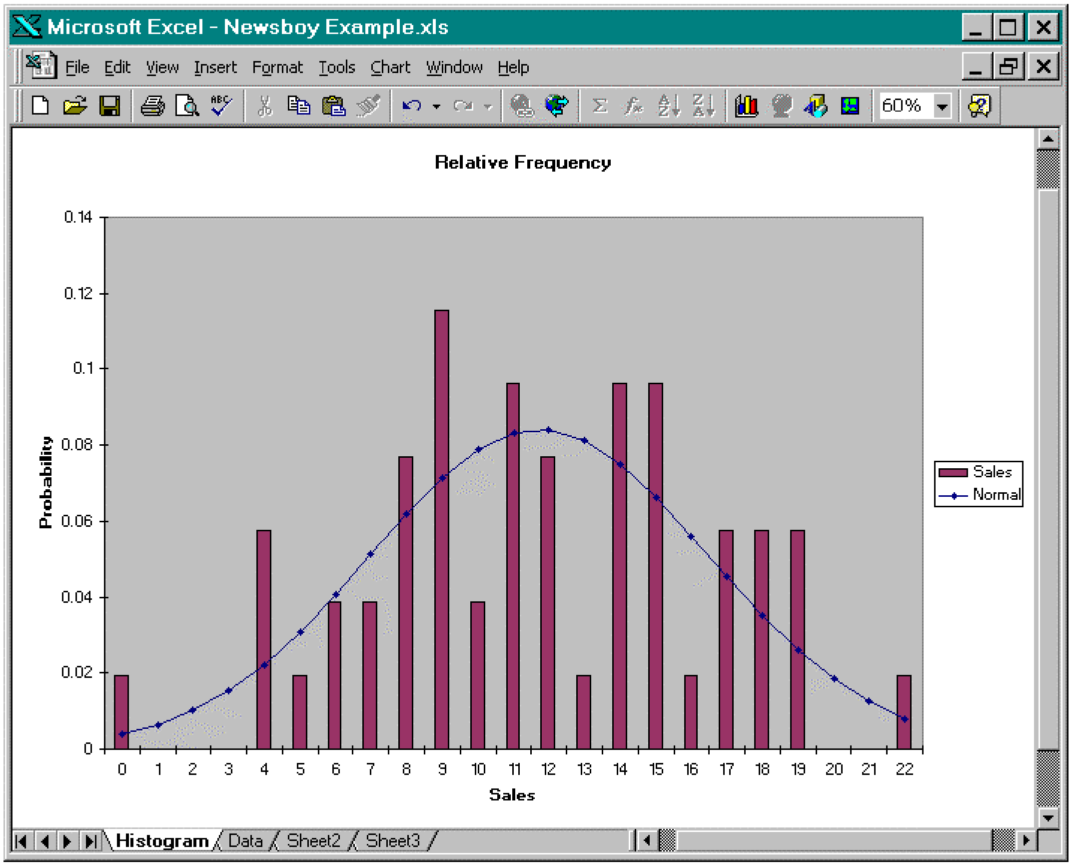
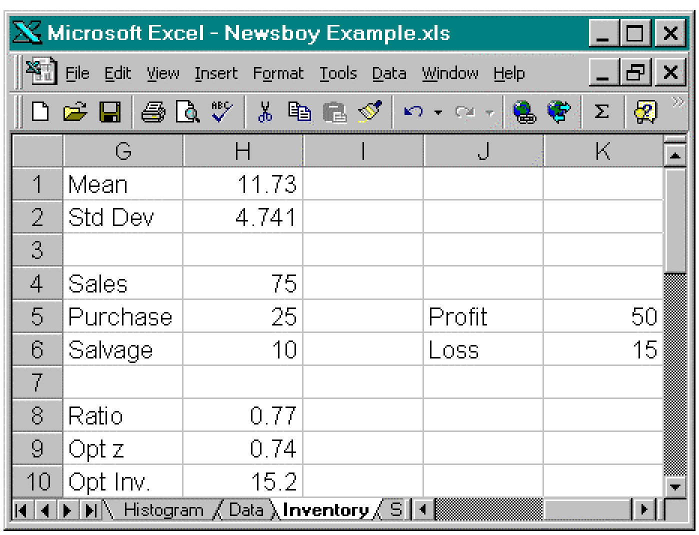


Figure 5.11. Histogram of Historical Sales



	G	H	I	J	K
1	Mean	11.73			
2	Std Dev	4.741			
3					
4	Sales	75			
5	Purchase	25		Profit	50
6	Salvage	10		Loss	15
7					
8	Ratio	0.77			
9	Opt z	0.74			
10	Opt Inv.	15.2			

Figure 5.12. Single Order Inventory Calculations

Safety Inventory

Customer Service Measured by Fraction Delivered from Inventory

Safety Inventory Commonly Equals Time Length Multiplied by Demand

If not demand uncertainty or stochastic demand then no safety inventory is required.

Continuous Review Policies

Sequential Determination of Q and R Based on Type 1 Service Level

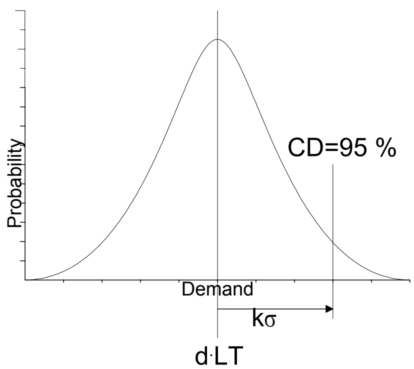


Figure 5.13. Demand During Lead Time Distribution

$$Q^* = \sqrt{\frac{2 \cdot D \cdot oc}{hc}}$$

$$s_{dlt} = s_d \sqrt{LT} \qquad (5.39)$$

$$R^* = d \cdot LT + z \cdot s_{dlt}$$

$$AI = CI + SI = \frac{Q}{2} + z \cdot s_{dlt} \qquad (5.40)$$

$$TC = \frac{D}{Q} oc + hc \frac{Q}{2} + hc \cdot z \cdot s_{dlt} = \frac{D}{Q} oc + hc \frac{Q}{2} + hc \cdot (R - d \cdot LT) \qquad (5.41)$$

$$Q = \sqrt{\frac{2 \cdot 200 \cdot 50}{0.2 \cdot 10}} = 100$$

$$s_{dlt} = \sqrt{0.5} \cdot 35.4 = 25$$

$$R = 200 \cdot 0.5 + 2.05 \cdot 25$$

$$= 151$$

$$TC = \left(\frac{100}{2} + (151 - 100) \right) 0.2 \cdot 10 + \frac{200}{100} 50 = 302$$

Sequential Determination of Q and R Using Shortage Costs

$$
\begin{aligned}
n(R) &= \int_R^\infty (x-R) f(x)dx \\
L(z) &= \int_z^\infty (t-z)\phi(t)dt \\
n(R) &= s_{dlt} L\left(\frac{R-d\cdot LT}{s_{dlt}}\right) = s_{dlt}\cdot L(z) \\
\frac{\partial n(R)}{\partial R} &= -(1-F(R))
\end{aligned}
\tag{5.42}
$$

The unit loss function $L(z)$ for positive standard deviates z is shown in the next figure.

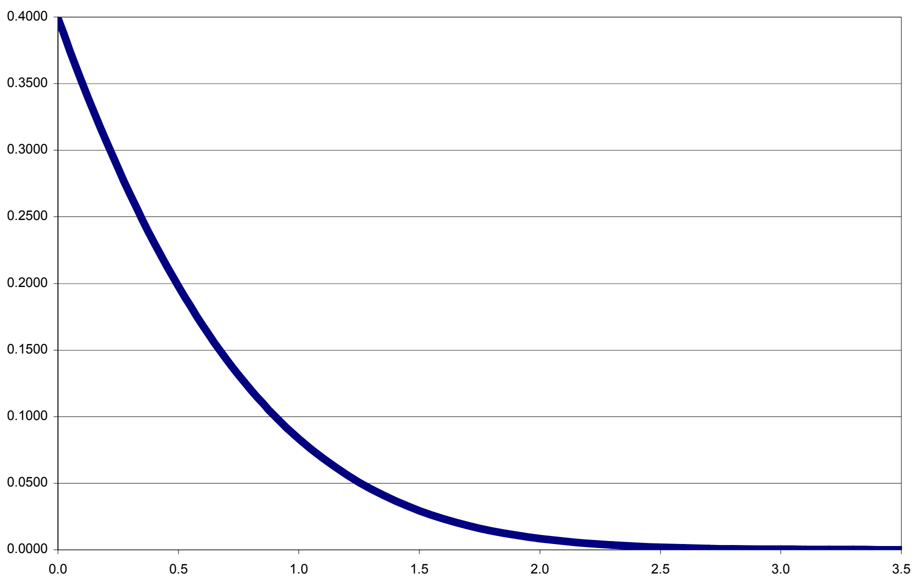


Figure 5.14. Unit Loss Function

$$
\begin{aligned}
TC(Q) &= \frac{D}{Q}oc + hc\frac{Q}{2} + hc\cdot z\cdot s_{dlt} + \frac{D}{Q}sc\cdot s_{dlt}\cdot L(z) \\
TC(Q,R) &= \frac{D}{Q}\left(oc + sc\cdot n(R)\right) + hc\left(\frac{Q}{2} + R - d\cdot LT\right)
\end{aligned}
\tag{5.43}
$$

$$
\begin{aligned}
TC &= \frac{200}{100}\left(50 + 25\cdot 25\cdot 0.0073\right) + \\
&= 10\cdot 0.2\left(\frac{100}{2} + 151 - 200\cdot 0.5\right) = 312
\end{aligned}
$$

Simultaneous Determination of Q and R Using Shortage Costs

$$\frac{\partial TC(Q,R)}{\partial R} = hc + \frac{D \cdot sc}{Q} \cdot \frac{\partial n(R)}{\partial R} = 0$$

$$\frac{\partial n(R)}{\partial R} = -(1 - F(R)) \qquad (5.44)$$

$$1 - F(R) = \frac{Q \cdot hc}{D \cdot sc}$$

$$\frac{\partial TC(Q,R)}{\partial Q} = \frac{hc}{2} - \frac{D(oc + sc \cdot n(R))}{Q^2} = 0 \qquad (5.45)$$

$$Q^* = \sqrt{\frac{2D\left[oc + sc \cdot n(R)\right]}{hc}} = \sqrt{\frac{2D\left[oc + sc \cdot s_{dlt} \cdot L(z)\right]}{hc}} \qquad (5.46)$$

The iterative procedure starts with the EOQ as initial value for the optimal order quantity.

$$Q^0 = \sqrt{\frac{2 \cdot D \cdot oc}{hc}} \qquad (5.47)$$

$$F(R) = 1 - \frac{Q \cdot hc}{D \cdot sc} \qquad (5.48)$$

$$z = F^{-1}\left(1 - \frac{Q \cdot hc}{D \cdot sc}\right) \qquad (5.49)$$

$$s_{dlt} = s_d \sqrt{LT}$$

$$R = d \cdot LT + z \cdot s_{dlt} \qquad (5.50)$$

$$Q = \sqrt{\frac{2D\left[oc + sc \cdot n(R)\right]}{hc}} = \sqrt{\frac{2D\left[oc + sc \cdot s_{dlt} \cdot L(z)\right]}{hc}} \qquad (5.51)$$

Microsoft Excel - Inventory EOQ Variants Harvey F...

	J	K	L	M	N	O
54	Q	F(R)	z	R	L(z)	n(R)
55	100.0	0.9600	1.7507	143.8	0.0161	0.4037
56	109.6	0.9561	1.7076	142.7	0.0179	0.4487
57	110.7	0.9557	1.7033	142.6	0.0181	0.4536
58	110.8	0.9557	1.7028	142.6	0.0182	0.4541
59	110.8	0.9557	1.7027	142.6	0.0182	0.4542

Book / Overhead / Sheet3

Figure 5.15. Inventory Example Iterative Algorithm for Type 1 Service Level

Sequential Determination of Q and R Based on Type 2 Service Level

Previously, the expected number of items backordered at the end of a single cycle was defined as $n(R)$. The expected number of cycles per year is equal to D/Q. The fill rate or long-range average ratio of the number of items delivered from inventory over the total demand is then equal to

$$fr = 1 - \frac{(D/Q)n(R)}{D} = 1 - \frac{n(R)}{Q} \tag{5.52}$$

In the sequential method, the economic order quantity is determined first. Based on the optimal order quantity and desired type 2 service level the reorder point is then determined.

$$\begin{aligned} Q &= \sqrt{\frac{2 \cdot oc \cdot D}{hc}} \\ L(z) &= \frac{Q(1-fr)}{s_{dlt}} \\ z &= L^{-1}\left(\frac{Q(1-fr)}{s_{dlt}}\right) \\ R &= d \cdot LT + z \cdot s_{dlt} \end{aligned} \tag{5.53}$$

The average inventory and total cost are then computed with the same formulas used for the type 1 service level.

$$AI = CI + SI = \frac{Q}{2} + z \cdot s_{dlt} \tag{5.54}$$

$$TC = \frac{D}{Q}oc + hc\frac{Q}{2} + hc \cdot z \cdot s_{dlt} = \frac{D}{Q}oc + hc\frac{Q}{2} + hc \cdot (R - d \cdot LT) \tag{5.55}$$

For the Harvey Foods example, the numerical values can then be computed as follows.

$$\begin{aligned} Q &= \sqrt{\frac{2 \cdot 200 \cdot 50}{0.2 \cdot 10}} = 100 \\ L(z) &= \frac{100(1-0.98)}{25} = 0.08 \\ z &= L^{-1}(0.08) = 1.02 \\ R &= 200 \cdot 0.5 + 1.02 \cdot 25 = 126 \end{aligned}$$

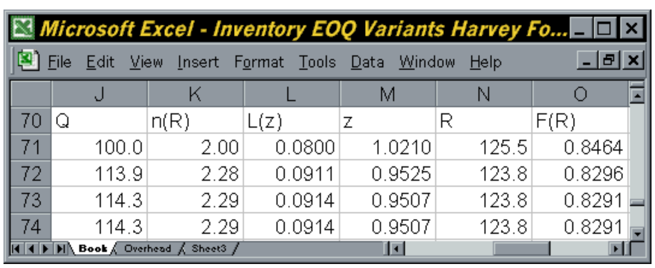
$$TC = \left(\frac{100}{2} + 1.02 \cdot 25\right)0.2 \cdot 10 + \frac{200}{100}50 = 251$$

Simultaneous Determination of Q and R Based on Type 2 Service Level

$$
\begin{aligned}
&Q^0 = \sqrt{\frac{2 \cdot oc \cdot D}{hc}} \\
&n(R) = Q\left(1 - fr\right) \\
&L(z) = \frac{Q(1-fr)}{s_{dlt}} \\
&z = L^{-1}\left(\frac{Q(1-fr)}{s_{dlt}}\right) \\
&R = d \cdot LT + z \cdot s_{dlt} \\
&Q = \frac{n(R)}{1-F(R)} + \sqrt{\frac{2 \cdot oc \cdot D}{hc} + \left(\frac{n(R)}{1-F(R)}\right)^2}
\end{aligned}
\tag{5.56}
$$

The numerical calculations for the first iteration of the Harvey Foods example are given next.

$$
\begin{aligned}
&Q = \sqrt{\frac{2 \cdot 200 \cdot 50}{0.2 \cdot 10}} = 100 \\
&L(z) = \frac{100(1-0.98)}{25} = 0.08 \\
&z = L^{-1}(0.08) = 1.0213 \\
&R = 200 \cdot 0.5 + 1.0213 \cdot 25 = 125.5 \\
&F(R) = F(1.0213) = 0.8464 \\
&Q = \frac{2}{1-0.8464} + \sqrt{\frac{2 \cdot 200 \cdot 50}{0.2 \cdot 10} + \left(\frac{2}{1-0.8464}\right)^2} = 113.9
\end{aligned}
$$



70	Q	n(R)	L(z)	z	R	F(R)
71	100.0	2.00	0.0800	1.0210	125.5	0.8464
72	113.9	2.28	0.0911	0.9525	123.8	0.8296
73	114.3	2.29	0.0914	0.9507	123.8	0.8291
74	114.3	2.29	0.0914	0.9507	123.8	0.8291

Figure 5.16. Inventory Example Iterative Algorithm for Type 2 Service Level

The values of Q and R do not show any significant changes at the end of the third iteration, so the iterative procedure terminates. The values of Q and R are then rounded to the nearest integer and the total cost is computed.

$$
TC = \left(\frac{114}{2} + 24\right) 0.2 \cdot 10 + \frac{200}{114} 50 = 249
$$

Stochastic Lead Time

$$
SI = k \cdot \sqrt{LT \cdot Var_d + d^2 \cdot Var_{LT}} \tag{5.57}
$$

$$CV_d = \frac{\sqrt{Var_d}}{d} \qquad (5.58)$$
$$Var_d = \left(CV_d \cdot d\right)^2$$

$$SI = k \cdot \sqrt{LT \cdot CV_d^2 + Var_{LT}} \cdot d \qquad (5.59)$$

The numerical values for the Harvey Foods example can then be computed as follows.

$$s_{dlt} = \sqrt{0.5 \cdot 35.4^2 + 200^2 \cdot 0.125^2} = 35.4$$

The iterative determination of the optimal Q and R parameters is shown in the next table.

Microsoft Excel - Inventory EOQ Variants Harvey Foods Exa...

	J	K	L	M	N	O
84	Q	n(R)	L(z)	z	R	F(R)
85	100.0	2.00	0.0566	1.1958	142.3	0.8841
86	118.7	2.37	0.0672	1.1106	139.3	0.8666
87	119.4	2.39	0.0675	1.1079	139.2	0.8660
88	119.4	2.39	0.0675	1.1078	139.2	0.8660
89	119.4	2.39	0.0675	1.1078	139.2	0.8660

Figure 5.17. Inventory Example Iterative Algorithm for Type 2 Service Level and Stochastic Lead Time

$$TC = \left(\frac{119}{2} + 1.11 \cdot 35.4\right) 0.2 \cdot 10 + \frac{200}{119} 50 = 282$$

The optimal values of the optimal order quantity and reorder point for the different inventory policies are summarized in the following figure.

Microsoft Excel - Inventory EOQ Variants Harvey Foods Example.xls

	A	B	C	D	E	F
1	Type	Q	R	SI	AI	TC
2	Deterministic Demand	100	0	0	50	200.0
3	+ Lead Time	100	100	0	50	200.0
4	Stochastic Demand	100	151	51	101	302.0
5	Shortage Cost (Seq)	100	151	51	101	311.2
6	Shortage Cost (Iter)	111	143	43	99	308.5
7	Type 2 Service (Seq)	100	126	26	76	252.0
8	Type 2 Service (Iter)	114	124	24	81	249.5
9	SL2 (Iter) + Stoc. Lead Time	119	139	39	99	281.5

Figure 5.18. Inventory Example Results Summary

5.3. Dependent Demand Systems

In dependent demand systems, the production and shipment of components and raw materials is timed to meet the production planning requirements. The right amount of material is produced and delivered at the right time. The goal is to avoid carrying items as work-in-process (WIP) inventory or to find the best balance between inventory holding cost and production setup costs. This inventory control policy is often used for the scheduling of the high-value, custom-made items. It requires that the demand for the finished product is reasonably well known or can be forecasted with relatively small errors.

Characteristics, advantages, and disadvantages of Distribution Requirements Planning

Distribution Requirements Planning or Distribution Resource Planning (DRP) uses the methodology and techniques of Material Requirements Planning (MRP) applied to the distribution channel to achieve integrated materials scheduling throughout the entire supply chain from raw materials suppliers to the finished goods customers. DRP is based on a centralized, top-down, push philosophy of supply chain management. The basic DRP methodology assumes that the future is known with certainty and demand is deterministic. Supply chains using the (Q,R) or (s,S) inventory policies generally manage products and retailers independently in a distributed, bottom-up, pull philosophy. These traditional reorder point policies explicitly incorporate uncertainty.

To a large extent, DRP will exhibit the same advantages and disadvantages of MRP. DRP has several advantages and benefits that derive from its centralized philosophy.

- DRP requires that similar and consistent data be collected throughout the entire supply chain. This in itself encourages integrated planning and enhances communication throughout the supply chain.
- DRP will show future shipments and deliveries of materials. This allows the management of the various facilities and transportation modes to anticipate their future requirements and enables them to better plan transportation, production, and warehousing activity levels.

- The requirements associated with new product introductions and sales promotions can be anticipated and the supply chain can be prepared for these sudden, irregular, or unusual requirements.
- When there is insufficient product inventory to satisfy all customer demand, the DRP system can allocate and distribute materials so that these shortages are more uniformly divided over the retailers. This prevents that one retailer may receive 100 % of its demand and another retailer receives nothing depending on which one transmitted their order first.
- DRP incorporates all sources of demand, not just the sales observed or transmitted by the retailer. For instance, the materials required for a seasonal inventory buildup are included and these requirements can be distributed over time.
- The system view of DRP allows the balancing of materials and requirements over the whole supply chain system. The utilization of distribution centers and manufacturing facilities can be balanced.
- The system-wide view allows also the obsolescence control of the goods in inventory. This is especially important in industry dealing in perishable products such as fresh food and flowers.

DRP has the following disadvantages, which are caused by its strong dependence on the quality of the forecast for the final products.

- Errors in the forecast or changes in the forecast may create work-in-process inventory of intermediate components in the supply chain.
- DRP considers only an uncapacitated system and manufacturing and transportation resource limitations are not considered or incorporated.

DRP Mechanics

DRP will compute required materials and orders backwards from the forecasted sale of the finished goods to the final customer. Based on given safety stock levels, lead times, shipping quantities, and the bill of materials (BOM), the forecasted sales are exploded backwards in time and through the supply chain. Forecasted demand is aggregated at the source facilities of the shipments.

The bill of materials gives the immediate components q required to produce one unit of product p. The BOM can be represented by a matrix $\mathbf{A}_{qp}$, where the rows represent the various components and the columns represent the various intermediate and end products. Element a_{qp} gives the number of units of component q required to produce one unit of product p. The following notation will be used:

i	on-hand inventory
d	gross requirements (outflow)
s	scheduled receipts (inflow)
r	planned production receipts (inflow)

The material requirements can then be computed with the following two equations. The first equation is a conservation of flow constraint for every commodity during every time period at every facility. The second equation translates the demand of a product a particular time into demand for its components at the earlier time periods determined by the lead time. The flow balance is illustrated in Figure (5.60)

$$i_{p,t} = i_{p,t-1} - d_{p,t} + s_{p,t} + r_{p,t} \tag{5.60}$$

$$d_{q,t-lt_p} = a_{qp} r_{p,t} \tag{5.61}$$

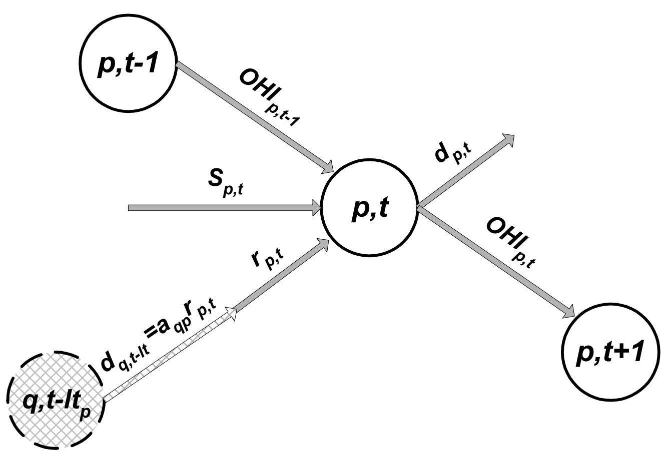


Figure 5.19. Flow balance at a Facility

The planned shipments at the various facilities and transportation modes are then used to generate the master production schedule and to update the safety stock levels and production and shipment quantity parameters. To schedule the production, the EOQ equation, the heuristic algorithm developed by Silver and Meal (1973), or the optimal dynamic programming procedure by Wagner and Within (1958) may be used. To incorporate uncertainty of the demand quantities, the newsvendor algorithm may be used at each stage to determine safety inventories in the supply chain. The use of safety stock levels is an extension to the basic deterministic DRP method.

DRP is a simple but effective tool for multilevel distribution systems or supply chains for products with complex multilevel bill of materials. Such systems typically include two or more layers or echelons of facilities that storing inventory. There exists an extensive theoretical literature on mathematical models for the management of multilevel inventory systems. However, most of these models are mathematically very complex or have significant simplifying assumptions that make their implementation and adaptation in large-scale industrial systems impractical.

MRP Example

M21	Week								
	0	1	2	3	4	5	6	7	8
Gross Demand		200	200	200	200	200	200	200	200
Receipts			600						
Quantity on Hand	500								
Production Start									

Figure 5.20. MRP Example M21 Model Data

There should be a production start at the beginning of week 1 for clock model M21.

M21	Week								
	0	1	2	3	4	5	6	7	8
Gross Demand		200	200	200	200	200	200	200	200
Receipts			600				600		
Quantity on Hand	500	300	700	500	300	100	500	300	100
Production Start		0	0	0	0	600	0	0	0

Figure 5.21. MRP Example M21 Model Schedule

	A	B	C	D	E	F	G	H	I	J
1	K36	Week								
2		0	1	2	3	4	5	6	7	8
3	Gross Demand		100	150	120	150	100	90	110	120
4	Receipts									
5	Quantity on Hand	400								
6	Production Start									

Figure 5.22. MRP Example K36 Model Data

	A	B	C	D	E	F	G	H	I	J
9	K36	Week								
10		0	1	2	3	4	5	6	7	8
11	Gross Demand		100	150	120	150	100	90	110	120
12	Receipts					350			350	
13	Quantity on Hand	400	300	150	30	230	130	40	280	160
14	Production Start		0	350	0	0	350	0	0	0

Figure 5.23. MRP Example K36 Model Schedule

	A	B	C	D	E	F	G	H	I	J
33	R1063	Week								
34		0	1	2	3	4	5	6	7	8
35	Service Parts		100	100	100	100	100	100	100	100
36	R36		0	350	0	0	350	0	0	0
37	M21		0	0	0	0	600	0	0	0
38	Gross Demand		100	450	100	100	1,050	100	100	100
39	Receipts									
40	Quantity on Hand	900								
41	Production Start									

Figure 5.24. MRP Example Component R1063 Data

	A	B	C	D	E	F	G	H	I	J
43	R1063	Week								
44		0	1	2	3	4	5	6	7	8
45	Gross Demand		100	450	100	100	1,050	100	100	100
46	Receipts					1,000	1,000			
47	Quantity on Hand	900	800	350	250	1,150	1,100	1,000	900	800
48	Production Start		0	1,000	1,000	0	0	0	0	0

Figure 5.25 . MRP Example Component R1063 Schedule

5.4. Exercises

True-False Questions

A common measure of customer service is the long-range fraction of product delivered out of inventory, (T/F) _____(1).

A fixed order quantity inventory control system is more suitable for small irregular demand than for regular, high quantity demand for a product, (T/F) _____(2).

A Just-In-Time Pull inventory system is best suited to regular demand patterns, (T/F)_____(3).

A lower inventory turnover ratio is desirable, (T/F)_____(4).

A mandated service level of 98 % of the product delivered out of inventory will create higher levels of inventory than the mandated service level of the probability of no stock out more than or equal to 98 %, (T/F) _____(5).

DRP is an extension of MRP to allow integrated supply scheduling throughout the entire logistics channel, (T/F)_____(6).

For a made-to-stock product the order cycle time for normal order fulfillment does not included the production time in the manufacturing plant, (T/F)_____(7).

For an inventory system with a stochastic lead time, the safety inventory in a system with a larger coefficient of variation of the lead time will be less than or equal to than the safety inventory in a system with a smaller coefficient of variation of the lead time, (T/F) _____(8).

If the coefficient of variation of the sample demand data is larger than one, then a normal distribution is a good approximation of the demand distribution, (T/F)_____(9).

If the cumulative demand for the mean plus two standard deviations is less than eighty percent then the normal distribution is a good approximation of the demand distribution, (T/F)_____(10).

If the future demand is known with perfect certainty, then there is no need to have inventory in the logistics system, (T/F)_____(11).

If the marginal loss is larger than the marginal profit in a single order inventory system, then the purchased inventory should be smaller than the expected demand, (T/F)_____(12).

If the marginal profit is larger than the marginal loss in a single order inventory system, then the purchased inventory should be larger than the expected demand, (T/F)_____(13).

In a pull inventory system the replenishments are based on the forecasted demand of the final and intermediate products, (T/F) _____(14).

In DRP the inventory levels of various components is dependent on the forecasted demand for the final finished good, (T/F) _____(15).

One of the main reasons to hold product inventory in the supply chain is to decouple the sequence of processing steps in the supply chain, (T/F) _____(16).

Response time constraints is one of the fundamental reasons to have warehousing and inventory, (T/F)_____(17).

Safety stock is the inventory held for speculation beyond the foreseeable needs of the corporation, (T/F)_____(18).

The aging or curing inventories used in the preparation of food products such as cheese and champagne can be modeled similar to pipeline inventory, (T/F) _____(19).

The average inventory caused by the iterative determination of Q and R for the case of stochastic demand and shortage cost in the system and with a service level constraint of type 1 will be less than or equal to the average inventory caused by the single pass determination of Q and R for the same case, (T/F) _____(20).

Tie Rack

	A	B	C	D
11	Purchase Price	p	0.11	$/unit
12	Lead Time	LT	1.5	months
13	Holding Cost Rate	hcr	0.01666667	/month
14	Shortage Cost	sc	0.01	$/unit
15	Ordering Cost	oc	10	$/order
16	Demand Rate	d	11107	units/month
17	SD Demand	sd	3099	
18	Probability No Stockout	F(R)	0.75	
19	Fill Rate	fr	0.75	
20	SD Lead Time	slt	0.5	months

Figure 5.26 . Inventory Tie Rack Exercise Data

Landgate

Landgate is a producer of a line of internal fixed disk drives for microcomputers. The drives use a 3.5-inch platter that Landgate purchases from an outside supplier. Demand data and sales forecast indicate that the weekly demand for the platters can be approximated with sufficient accuracy by a normal distribution with mean of 3800 and variance of 1,300,000. The platters require a three-week lead-time for receipt at the disk drive assembly plant. Landgate has been using a 40 percent annual interest charge to compute the holding costs because of very short life cycle of the disk platters, since platters with higher data density are released frequently. The platters cost $18 each and the ordering cost is $3700 per order, which includes the transportation cost to the assembly plant by airfreight. Because of the extreme competitiveness of the industry, stock-outs are very costly and Landgate uses currently a 99 percent fill rate criterion. You have been asked to determine the inventory parameters for a continuous review inventory policy that minimizes the total cost associated with the inventory while satisfying the customer service constraints. First, give the numerical value of the problem parameters. Then compute the inventory policy parameters with the single-iteration algorithm. Next compute these parameters with the appropriate iterative algorithm, but compute only a single iteration starting from the solution of the single-pass algorithm. Specify the units for all variables. Round all numerical values related to platters (e.g. demand, standard deviation, and inventory) to the nearest integer number of platters. You can assume that there are 52 weeks in a year. Briefly discuss and explain the changes in average inventory and total cost between the one-pass and iterative algorithm.

Christmas Cards

The Deck-The-Halls (DTH) Printing Company once a year prints a particularly richly decorated and ornate Christmas card and distributes the cards to department stores, and stationary and gift shops throughout the United States. It costs DTH $2.50 to print each card and ship the card to a shop. The company sells the cards for $12.50 a piece, which includes transportation. Each shop or store receives a single order before the start of the Christmas season. Because the cards have the current year printed on them and DTH wants to maintain the image of this particular signature card series, those cards that are not sold to the shops are destroyed. Based on past experience and forecasts of the current buying patterns, the probabilities of the number of cards to be sold nationwide for the coming Christmas season are estimated to be as shown in the next table.

Lower Bound Interval	Upper Bound Interval	Interval Probability
100000	150000	0.0625
150001	200000	0.1250
200001	250000	0.1875
250001	300000	0.2500
300001	350000	0.1875
350001	400000	0.1250
400001	450000	0.0625

Determine the number of cards that DTH should print this year. Summarize your calculations in a table, using a row for the computation of each relevant variable. In the first column, give the name of the variable you are computing, in the second column give the formula used to compute the variable, in the third column give the numerical value, and in the fourth column give the units of the variable. You cannot receive credit for the numerical value unless the units are specified.

A detailed analysis of the past data shows that the number of cards sold is better described by a normal distribution, with mean of 275,000 cards and standard deviation of 80,000 cards. Based on this sales forecast how many cards should DTH print now? Again summarize your calculations in a table, using the same conventions.

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